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MAGNET SYSTEM FOR THE PLANCK-BALANCE

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ABSTRACT

The current definition of the unit kilogram by an artifact is about to be replaced by a definition based on a fundamental constant, which is the Planck constant. To establish the link between a mass standard and the Planck constant an instrument called Watt- or Kibble-Balance can be used. This instrument uses a virtual equilibrium between mechanical and electrical power to connect the mass to the electrical quantities which can be defined from the Planck constant using the Josephson- and the Quantum-Hall effect. After the redefinition of the unit, the new definition can be realized not only for masses of 1 kg but for any arbitrary mass value. Consequently the new definition can be directly applied to weighing processes in industry if a suitable measurement system exists.
In this paper we briefly describe the design of such a weighing system, called the Planck-balance PB2 which is designed to realize the new definition in a mass range from 1 mg to 100 g. In accordance to our theoretical estimations we aim for achieving uncertainties comparable to current mass disseminations using calibrated weights of class E2. The magnet system is a crucial part of the PB2, since it is responsible for compensation of the weight of up to 100 g with a minimal amount of power loss and because it needs to be characterized with relative uncertainties in the sub ppm range. This article is dedicated to the design considerations for the magnet system which were done using an analytical and a numerical approach. The results are compared to initially obtained measurement data of a prototype setup.

Index Terms - Force measurement, Watt balance, Planck constant, Mass dissemination

1. INTRODUCTION

After the redefinition of the kilogram which is planned for the year 2018 the numerical value of the Planck constant will be fixed and the kilogram can be determined by using a Kibble balance. Kibble balances are used to establish a link between the Planck constant and the mass m by means of a virtual comparison of electrical power which is determined from the product of voltage U and current I and the mechanical power determined from the product of mass m, gravitational acceleration g and the velocity v [1].

$$m\, g\, v = UI \tag{1}$$

The electrical power can be expressed using the Planck constant h and the conventional values for the Josephson constant K_{J-90} and the von Klitzing constant R_{K-90} according to [1] by

$$UI = \frac{U_{90}U_{90}'}{R_{90}}\frac{K_{J-90}^2 R_{K-90}}{4}h \tag{2}$$

Where U_{90} is the conventional voltage measured in the force mode, U_{90}' the conventional voltage measured in the velocity mode and R_{90} the conventional Resistance value of the measurement resistor used in the force mode. Currently most of the Kibble balances operated by the National metrological institutes are used to determine the numerical value of the Planck constant required for redefinition using a mass of 1 kg. For those measurements uncertainties in the range of 1e-8 are reported [2, 3]. The currently existing Kibble balances in National Metrological Institutes are developed primarily for the purpose of the redefinition and are designed to operate with masses of about 1 kg. They achieve high precision measurements with low uncertainties but need large effort and investment to be operated [4]. For example a measurement cycle for one value of the Planck constant using the Kibble balance of the National Research Council (NRC) Canada requires 1.5 h [3].
The mass range where the Kibble balance technique can be applied is not limited to the range of about 1 kg and can be expanded to a wider range. This provides the advantage that smaller masses can be calibrated directly without mass comparisons between weights to scale down the mass from 1 kg to smaller values, thereby avoiding uncertainty contributions and effort of multiple mass comparisons. Additionally, besides of the calibration of standard mass pieces such a device could be directly used for arbitrary mass measurements, meanwhile avoiding the necessity of reference weights for calibration of the balance before a measurement.

2. CONCEPT OF THE PLANCK-BALANCE

The concept is aimed at achieving typical uncertainties in industrial mass calibration which are defined by the requirements of the OIML R 111-1 standard [5] at fraction of the costs and effort required for operating the Kibble balances used for redefinition. This allows the Planck-balance to be used by national metrology institutes as well as industrial companies, pharma industry and other fields where traceable mass calibrations are required. Additionally the instrument should have a size comparable to a standard laboratory balance that it can be used as a tabletop system. The international standard OIML R 111-1 defines different classes of weights where E1 represents the class with the lowest permissible measurement uncertainty. The subsequent class with higher permissible uncertainty is called E2. Related to this convention the Planck-balance will be available in two versions (PB1 and PB2) with two measurement ranges and with two different levels of permissible measurement uncertainty. The measurement range of the PB1 from 1 mg to 1 kg is shown with blue transparent background and the measurement range of the PB2 from 1 mg to 100 g is shown with red transparent background in Figure 1.

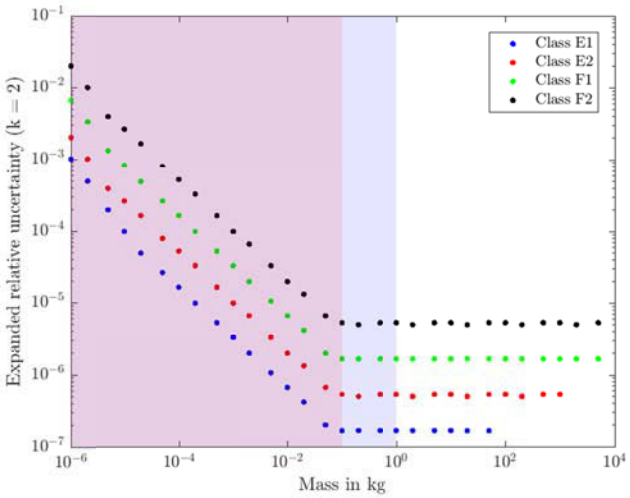

Figure 1: Uncertainties of weights of the classes E1, E2, F1 and F2 according to OIML R111 and measurement range of the Planck-Balances PB1 and PB2

To achieve those specifications the PB1 needs to be operated in vacuum, while the PB2 can be used in air under laboratory conditions. In accordance to the operation of the Kibble balance, the Planck-balance also uses two measurement modes – a velocity mode (A) and a force mode (B), which are used in a periodical manner (ABBA) for canceling out effects by linear drifts of measurement devices. Using this principle of repeated calibrations and measurements, the time required for a mass calibration using the Planck-balance is in the range of 10 to 120 s. The Planck-balance is based on the principle of the Kibble balance and consists of an electromagnetic force compensation balance (EMFC balance) [6,7] which is equipped with an additional magnet system and an interferometer for precise velocity measurement and position control in the force mode.

3. OPERATION OF THE PLANCK-BALANCE

The Planck-Balance measurement is based on establishing a virtual equilibrium of electrical and mechanical power (eq. (1)). It is called a virtual equilibrium because not all measured quantities are present at the same time, but are determined in two sequential measurement modes, called velocity mode and force mode. Those two modes of operation lead to the two separate equations

$$U_v = Bl \cdot v, \text{ and} \tag{3}$$

$$m \cdot g = Bl \cdot I = Bl \cdot \frac{U_f}{R} \tag{4}$$

which are combined to determine the mass m from voltage (U_v, U_f), resistance R, velocity v and gravitational acceleration g. The diagrammatic representation of the measurement system is shown in Figure 2 including standards for voltage, resistance, velocity and gravitational acceleration to realize the unit of mass directly from them. If the gravitational acceleration is unknown the Planck-Balance can serve as a force standard to realize the Newton directly traceable to the Planck constant.

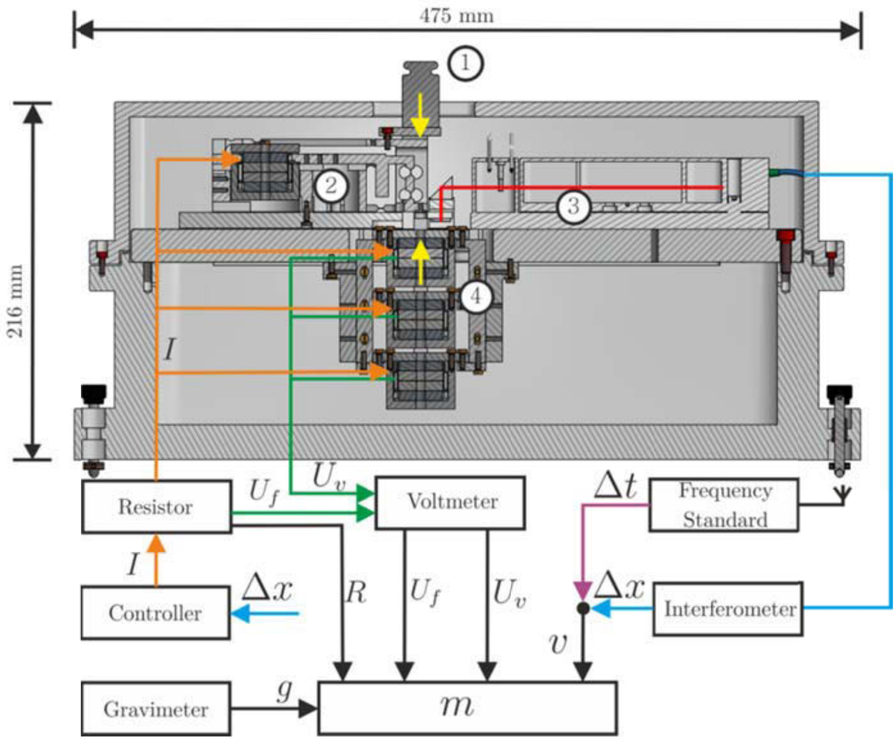

Figure 2: Overview of the components and the measurement scheme of the PB2 with a weight to be calibrated (1), EMFC balance (2), Interferometer (3) and combined magnet system from three single magnet systems (4)

In the velocity mode the mass (1) is removed from the weighing pan of the balance (2) and the coils (4) attached to the balance are oscillated in the magnetic field generated by their magnet systems (4). During the velocity mode the velocity v of the coil motion is measured using an interferometer (3) and the induced voltages in the coils U_v are measured using a precision voltmeter to determine the calibration values of the magnet systems Bl from eq. (3).
During the force mode the mass (1) is placed on the weighing pan of the balance (2) and the magnet systems and coils (4) which were calibrated in the velocity mode are used to generate a force which balances the weight to be calibrated (yellow arrows in Figure 2). When the forces are brought to equilibrium by means of a control system that uses the interferometer signal as an input eq. (4) holds and the mass can be determined by combining eq. (3) and eq. (4) in the following form

$$m = \frac{U_v U_f}{v \cdot g \cdot R}. \tag{5}$$

The relative measurement uncertainty of using a Planck-Balance for mass determination can roughly be estimated by eq. (6) which can be derived from eq. (5) by standard uncertainty propagation. The uncertainty of the mass depends on the relative measurement uncertainties of voltage, velocity, resistance and gravity measurements.

$$\frac{u(m)}{m} = \sqrt{\left(\frac{u(U_v)}{U_v}\right)^2 + \left(\frac{u(U_f)}{U_f}\right)^2 + \left(\frac{u(v)}{v}\right)^2 + \left(\frac{u(g)}{g}\right)^2 + \left(\frac{u(R)}{R}\right)^2} \tag{6}$$

The voltage measurements in the PB2 are done using a Keysight 3458A-002 multimeter. For the measurement of U_f and U_v in the measurement range of 10 V with a temperature deviation of $\pm$ 1 K from calibration temperature and within 24 h from calibration, the relative uncertainty can be approximated with 0.55 ppm [8].
The velocity measurement with the interferometer is done by length and time measurements. Uncertainties in the length measurement result from the frequency stability of the laser and from the temperature, humidity and pressure dependent refractive index if the measurement is done in air [9]. For our system the relative uncertainties of the velocity measurement are approximated with 0.1 ppm.
The value of the gravitational acceleration at the measurement site was determined by the Bundesamt für Kartographie und Geodäsie (German Federal Agency for Cartogrphy and Geodesy) to $\mathrm{g} = 9.81017160 \pm 0.17 \cdot 10^{-6}\,\mathrm{ms}^{-2}$. This value has temporal variations (mainly due to tides) of $\pm 2 \cdot 10^{-6}\,\mathrm{ms}^{-2}$ which are assumed to be uncorrected for this uncertainty estimation. The relative uncertainty from those influences is approximately 0.2 ppm. Resistors with calibration uncertainties of 0.3 ppm are commercially available. Their temperature coefficient is given with 0.5 ppm K^{-1} [10].
The combination of those uncertainty contributions using eq. (6) leads to an estimated standard uncertainty of the mass measurement of $\mathrm{u(m)/m} = 0.86$ ppm. As can be seen from Figure 1 this value is sufficient for E2 mass calibrations of weights ≤ 10 g and E1 calibrations of weights $\leq$ 2 g. To extend the range to higher masses the uncertainty needs to be reduced. This can be done by reducing the calibration uncertainties of the multimeters and resistors through shorter calibration intervals and drift corrections as well as correcting temporal variations in the gravity measurements.

4. MAGNET SYSTEM

The magnet system is an important component of the Planck-Balance because it influences the Bl factor which occurs in both modes of operation. The establishment of the virtual equilibrium of electrical and mechanical power is based on the assumption that Bl in eq. (3) and eq. (4) is identical. In the Planck-Balance both modes are not carried out simultaneously like in [11], therefore Bl should be stable in the sub ppm range to achieve the desired uncertainties.

To reduce deviations between the two measurement modes the power loss in the coil of the magnet system during the force mode needs to be kept small to reduce effects like thermally introduced changes of the magnetic field and mechanical changes due to thermal expansion of the components. The power required for generating a Lorentz force with a magnet system is

$$P \approx \frac{F^2}{\sigma\, B^2\, V}. \tag{7}$$

To reduce the power required for compensating a force F the electrical conductivity σ, the magnetic flux density in the air gap B and the coil volume V of the coil could be increased. The conductivity is limited by the available materials that can be used to manufacture the coil. In the Planck-Balance the coil is made of copper wire with an electrical conductivity of $\sigma = 6 \cdot 10^7\ \mathrm{Sm^{-1}}$. The magnetic flux density is limited by the Remanence of the available permanent magnet materials. Currently the material with highest remanence available is NdFeB. The drawback in using NdFeB is its high temperature coefficient of Remanence which is approximately $-1000\ \mathrm{ppm\,K^{-1}}$. Instead of using NdFeB we chose SmCo which has a Remanence that is about 20% lower than NdFeB but has only about one third of its temperature coefficient value. From eq. (8) (details on its derivation can be found in [12]) the magnetic flux density in the air gap of the magnet system can be calculated based on the magnet system parameters given in Figure 3

$$B(r) = \frac{B_r h_m ({r_{ma}}^2 - {r_{mi}}^2)}{r\left(2\, h_m\, h_p + \mu_r\{{r_{ma}}^2 - {r_{mi}}^2\}\ln\{r_{la}/r_{li}\}\right)}. \tag{8}$$

The flux density generated by the magnet in the air gap for a fixed ratio of the magnet height to the magnet radius (h_m/r_{ma}) is independent from the magnet radius while the coil volume is increasing with increasing magnet radius. By increasing the magnet system radius the power loss can be reduced with $P \propto r^{-1}$ which is a reason for using large magnet systems in the Kibble balances.

For a tabletop system like the Planck-Balance, magnet systems of comparable size like in the Kibble balances are not applicable. If the magnet system dimensions have to be kept small, the number of magnet systems applied to compensate the force can be used for reducing the power loss. If n equal magnet systems are used, the force per magnet system scales with $F \propto n^{-1}$. Due to the quadratic dependency of the power on the force, the power per magnet system scales with $P \propto n^{-2}$. In the Planck-Balance we use three coils and magnet systems which is an acceptable compromise between reduction of power loss and effort for alignment and electrical connections for all coils. The additional magnet systems provide the advantage, that they can be used to calibrate each other vice-versa. One coil is used for generating the force required for moving the mechanical system with the velocity v while the induced voltage is measured at the other two coils. Then the coils are switched and the induced voltage is measured at the coil which was used to drive the velocity mode first. After the velocity mode experiments all coils can be used to generate a combined force in the force mode experiment.

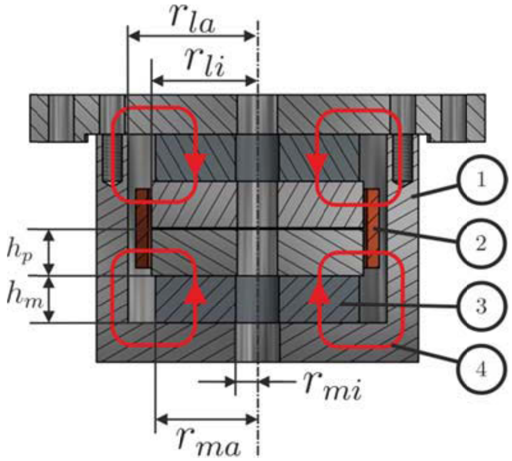

Figure 3: Dual magnet system used in the Planck-Balance consisting of a ferromagnetic flux guide (1), a coil (2) and two permanent magnets (3) which are generating a radial magnetic field in the air gap

From Figure 3 we can see that the magnet systems used in the Planck-Balance have a similar magnet configuration like the magnet systems of Kibble balances [10, 11]. They are made from two permanent magnets with opposing magnetization indicated by the red arrows in Figure 3. This design concentrates the magnetic flux density in the air gap and reduces stray fields. A ferromagnetic flux guide is used to focus the flux in the area of the coil and shield the magnet systems from external influences. The magnetic flux density of the magnet system is an important design parameter since it influences the *Bl* factor of the Planck-Balance and the power loss according to eq. 7. For a quick estimation of the flux density eq. (8) can be used to calculate the radial component of the flux density inside the air gap.
For comparison, the magnetic flux density inside the air gap is calculated using a 2D magnetostatic simulation in COMSOL. From Figure 4 right we can see that the deviations between the analytical and numerical calculations are below 10 %. After further refinement the analytical model could be used for fast calculations that could be required for design optimization or uncertainty estimation from design parameter variations (e.g. change of remanence or geometrical dimensions caused by temperature changes) using Monte-Carlo Methods.

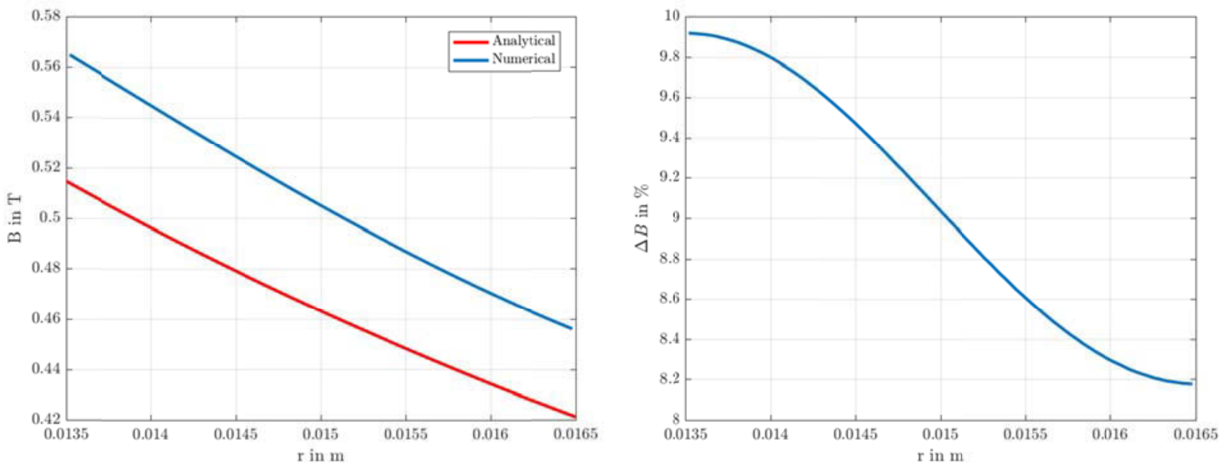

Figure 4: Radial magnetic field in the air gap of a PB2 magnet system calculated analytically and numerically (left) and relative deviation between analytical and numerical calculation (right).

From the results we can see that the magnetic field magnitude in the air gap is comparable to the flux density achieved in the Kibble balances [12, 13].

Additional to the radial field distribution *Bl* as a function of the axial coil position was calculated numerically and shown in **Fehler! Verweisquelle konnte nicht gefunden werden.** left for a coil motion range of $\pm$ 3 mm. Because the coil of the Planck-Balance has only a motion range of $\pm$ 0.1 mm, the alignment of the coil to the flat center domain of the *Bl* curve is crucial for optimally using the magnetic field provided by the magnet system.
Through the magnetic field generated by the current in the coil, the magnetic field of the permanent magnets is distorted. Because *Bl* is measured in the velocity mode without a current through the coil this can lead to a deviation of *Bl* between the force- and the velocity modes shown in **Fehler! Verweisquelle konnte nicht gefunden werden.** right. For a coil which is ideally centered in the magnetic field during the force mode the deviation is zero. If the coil is generating one third of the force required to compensate the mass of 100 g (≈ 0.33 N) it needs to be aligned within 0.5 µm to the ideal coil position if a relative deviation below 0.1 ppm is required. The ideal coil position can be identified during the velocity mode and used in the position control of the force mode for adjustment of the coil position.

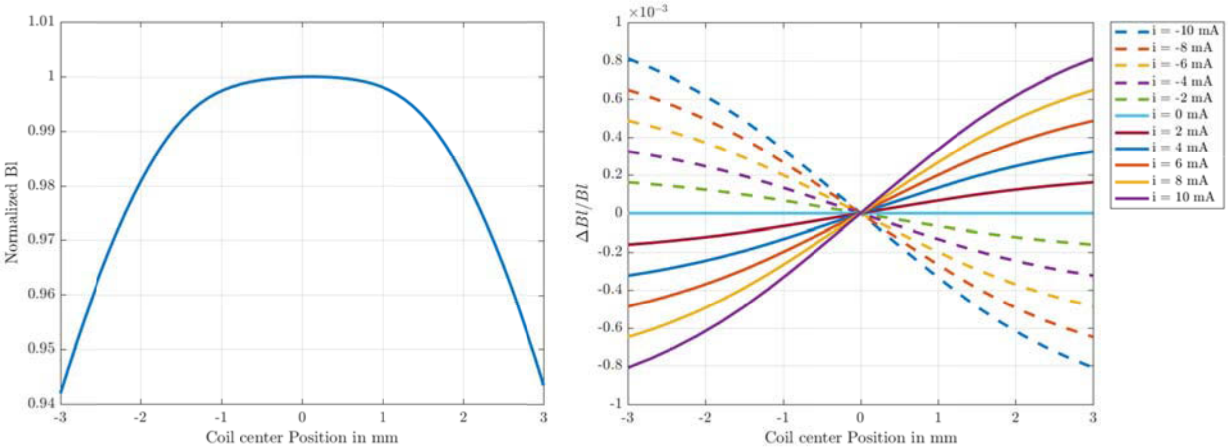

Figure 5: Bl as a function of the axial coil position (left) and deviations of Bl as a function of the axial coil position for different currents through the coil in the force mode of the Planck balance (right).

To check the results of these numerical simulations initial measurements of the *Bl* factor were done on a magnet system and coil. The measurement was done by changing the axial position of the coil while compensating a constant force and measuring the coil current. The results presented in Figure 6 show a relatively good agreement of the numerical simulations with the initially made rough measurements.

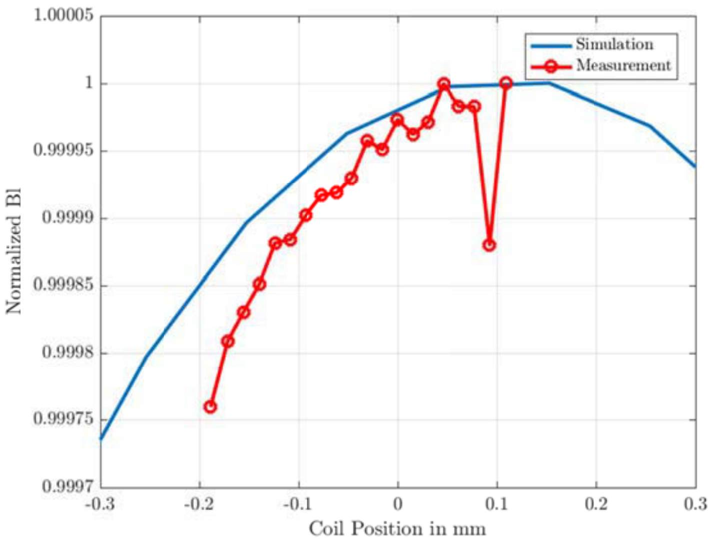

Figure 6: Comparison of Bl normalized with the maximum value from measurements and numerical simulations.

Another effect related to the coil current is the magnetic stiffness k_R which causes parasitic reluctance forces if the coil is not operated in the center position of the magnet system. As described in [14] a magnetic stiffness can be derived from the analytical model of [12] with the magnet system design parameters from Figure 3 as well as the current I and the number of windings N of the coil.

$$k_R = -\frac{\pi\,\mu_0\,N^2 I^2\,(r_{li}+r_{la})}{h_p(r_{la}-r_{li})}. \tag{9}$$

For the magnet system and coil parameters of the existing Planck-Balance setup the magnetic stiffness is $k_R = -0.24\ \mathrm{Nm}^{-1}$ for compensating one third of the mass of 100 g. The required positioning uncertainty of the coil to reach 0.1 ppm maximum deviation of the coil position is 0.13 µm. This positioning accuracy can be achieved by the position control loop in the force mode if the interferometer signal is used as an input signal for the control loop.

5. SUMMARY AND OUTLOOK

In this paper we introduced a weighing instrument called the Planck-Balance which is realizing the new definition of the kilogram based on the Planck constant and can be used in industrial environment. The device will be developed in two versions which are labelled as PB1 and PB2 according to the weight classes E1 and E2. The setup of the PB2 and its characteristic two modes of operation, known as velocity mode and force mode, are described. From a preliminary uncertainty analysis using datasheet specifications of the PB2 components a relative uncertainty of 0,86 ppm is estimated for the mass determination.

The magnet system of the Planck-Balance is an important component as it influences the *Bl* factor which occurs in both modes of operation. From the required equality of *Bl* in the force and velocity mode the considerations for the arrangement of magnet systems are derived.

The magnet systems are designed as dual magnet systems to achieve a strong magnetic field in the air gap while reducing stray fields and external disturbances. Analytical and numerical computations are made to estimate the magnetic flux density inside the air gap with deviations below 10% between both approaches. In future the models will be refined to cover several specific details of the our application.

Additionally the numerical model is used to calculate *Bl* as a function of the axial coil position and to estimate sources of deviations from *Bl* between the force and velocity mode arising from the magnetic field generated by the current in the force mode. To check the results of the numerical calculation they are compared to preliminar measurements showing an agreement within 0.015%.

Further sources of measurement deviations between the force- and the velocity mode will be investigated numerically and used for new magnet system designs for the two versions of the Planck-Balance.

6. ACKNOWLEGEMENT

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